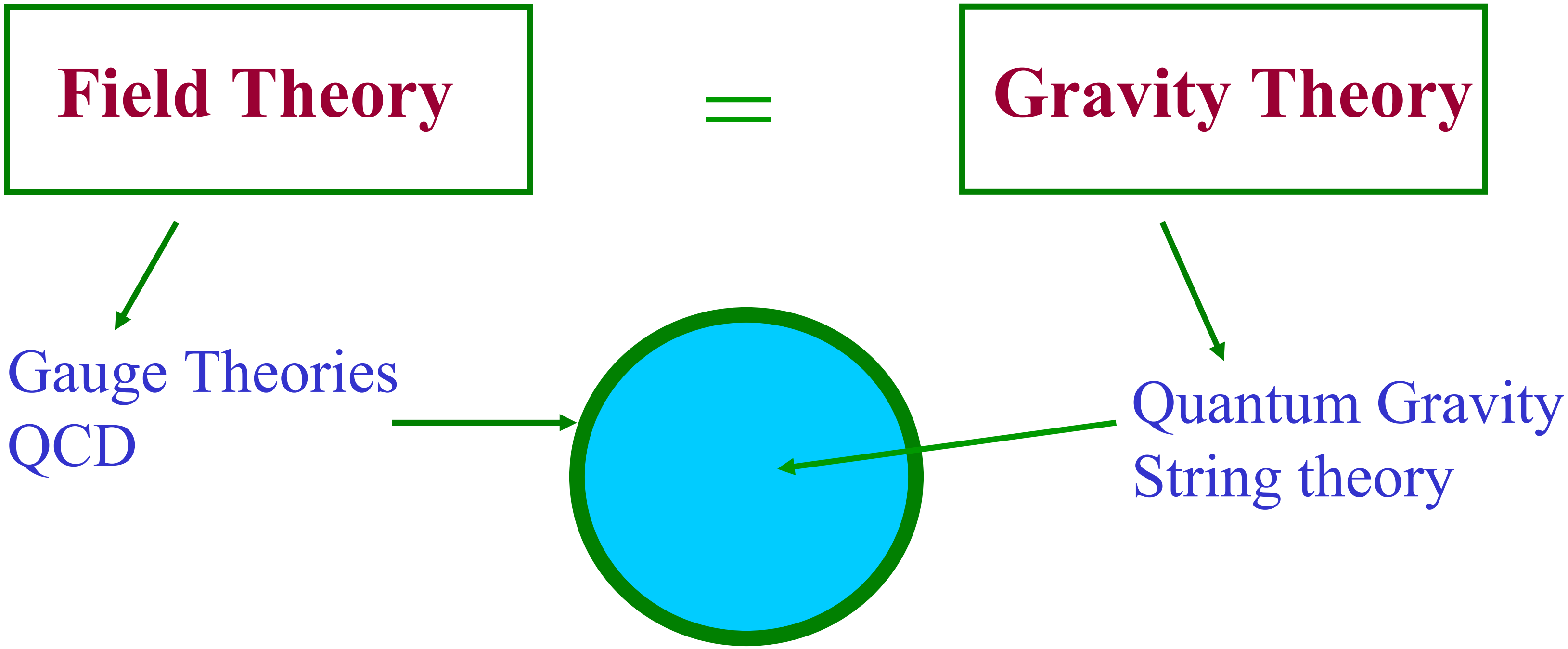
Quantum Gravity As an Ordinary Gauge Theory

Juan Maldacena
Institute for Advanced Study
Princeton, New Jersey



PLAN:

Physics of anti-de-Sitter spacetimes

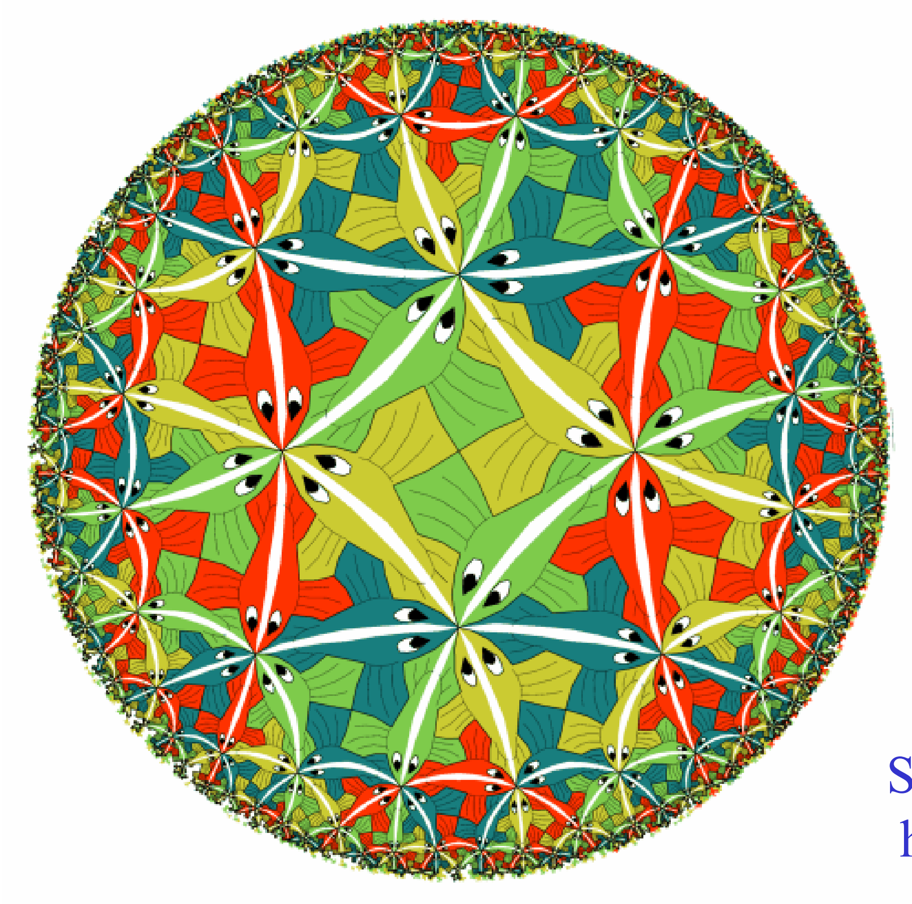
Gauge theory on the boundary

Plane wave limit of the correspondence

Anti de Sitter Space

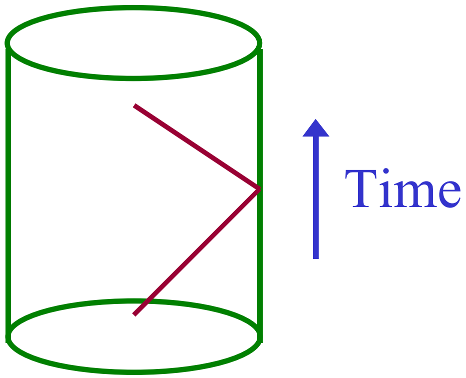
Solution of Einstein's equations with negative cosmological constant.

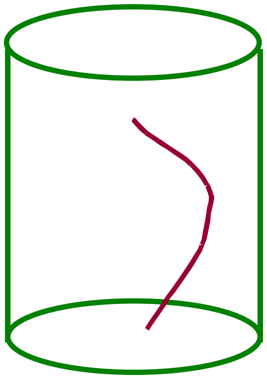
(De Sitter → solution with positive cosmological constant)



Spatial cross section of AdS = hyperbolic space.

Penrose Diagram:





Energies of particles in AdS are quantized, particles feel as if they were in a potential well, they cannot escape to infinity.

R = radius of curvature

Isometry group = SO(2,d-1) for AdS_d

Boundary is S^3 x Time (for AdS_5)

SO(2,4) maps the boundary to itself and it acts on the boundary as the conformal group in 3+1 dimensions. $S^3 = R^3 +$ (infinity)

The Field theory is defined on the boundary of AdS.

Since AdS has a boundary, we need some boundary conditions.

Physics in the interior depends on the boundary conditions.

What are the observables in the full quantum theory?

In a general setting the observables of quantum gravity are not known.

For spacetimes with suitable asymptotics they can be defined.

For asymptotically flat spacetimes the S-matrix defines an observable.

For asymptotically AdS spacetimes one can define the full partition function of the theory as a function of the boundary conditions.

By changing the boundary conditions we can send in particles to the interior or extract particles.

The correspondence

Quantum gravity on AdS_D	$=$	Local quantum field theory on the boundary of AdS

The partition function with given boundary conditions in the gravity theory is the same as the partition function of the quantum field theory. Different boundary conditions on the gravity side translate into different Lagrangians for the field theory.

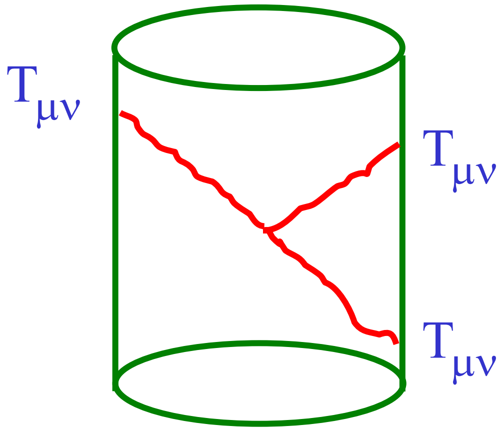
The states in AdS correspond to states in the field theory.

The field theory is defined on S^{D-2} x (Time) , so it will have a discrete spectrum. The spectrum of energies in AdS is the same as the spectrum of energies in the dual field theory.

("Local" QFT, means that the quantum field theory has a local stress tensor.)

The theory in the bulk (in AdS) contains gravity

Graviton $\longrightarrow$ stress tensor



Gubser, Klebanov, Polyakov - Witten

$< T_{\mu\nu}(x)\ T_{\mu\nu}(y)\ \ T_{\mu\nu}(z) >_{\text{Field theory}}$ = Probability amplitude that gravitons go between given points on the boundary

Other operators

$\longrightarrow$ Other fields (particles) propagating in AdS.

Mass of the particle $\longrightarrow$ scaling dimension of the operator

$$\Delta = 2 + \sqrt{4 + (mR)^2}$$

Example:

$\mathcal{N} = 4$ SU(N) Yang-Mills theory = String theory on $AdS_5 \times S^5$ (J.M.)

Radius of curvature $$R_{S^5} = R_{AdS_5} = \left(g_{YM}^2 N\right)^{1/4} l_s$$

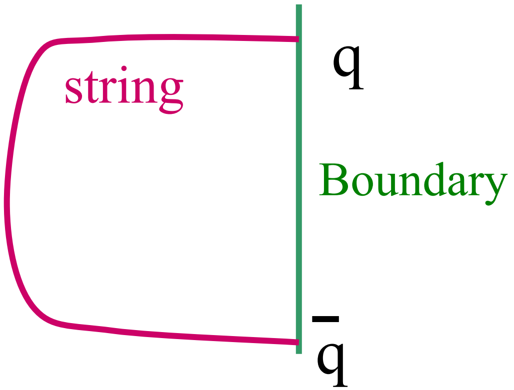
Duality:

g^2N is small ⟶ perturbation theory is OK

g^2N is large ⟶ Gravity is a good description of the system.

Strings made with gluons become fundamental strings.

Quark Anti-Quark Potential



V = potential = proper length of the string in AdS

(J.M., Rey, Yee)

$$V \approx \frac{\sqrt{g^2 N}}{L}$$

This is the correct answer for a conformal theory, the theory is not confining.

The reason that we get a small potential at large distances is that the string is goes into a region with very small redshift factor.

Baryons $\longrightarrow$ D-branes

Where Do the Extra Dimensions Come From?

3+1 ⟶ AdS_5 ⟶ radial dimension

Boundary

z

$1/z \sim$ energy scale

z=0

z=infinity

$$ds^2 = \frac{dx_4^2 + dz^2}{z^2}$$

ultraviolet,
high energies

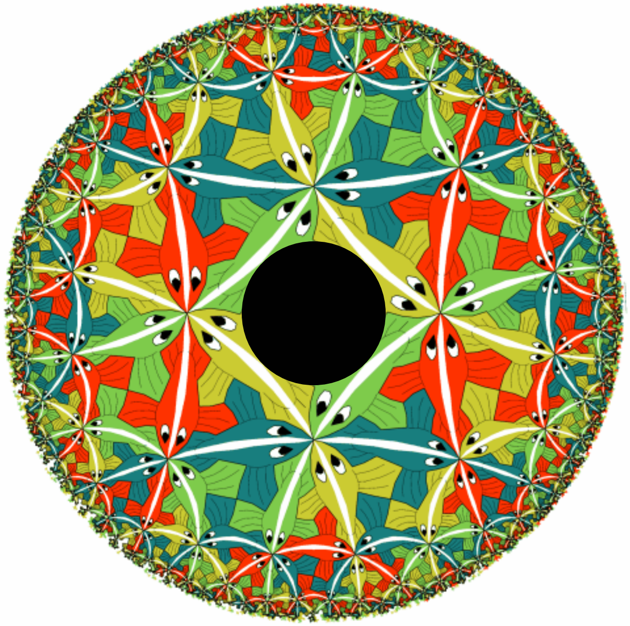
infrared,
low energies

Renormalization group flow: Motion in the radial direction

Five-sphere: is related to the scalars and fermions in the supersymmetric Yang-Mills theory. For other theories the sphere is replaced by other manifolds, or it might even not be there.

Black holes in AdS

Thermal configurations in AdS.
One can calculate the entropy in the field theory and it agrees with the gravity result.
(these calculations are easier in the AdS_3 case)



D-brane models of black hole in string theory involve a duality between AdS_3 and a 1+1 dimensional conformal field theory.

Strominger, Vafa
Callan, JM
Das- Mathur
……

Black hole phase transitions

AdS at finite temperature $\longrightarrow$ Two states

Witten

1. Gas of particles in AdS $\quad T < T_c \quad S \sim o(1)$

2. Black hole in AdS $\quad T > T_c \quad S \sim 1/G_N \sim N^2$

$\downarrow$

Like a deconfinement phase transition

Properties of the black hole can be used to get some insight on the dynamics of high temperature QCD, and the dynamics of the quark gluon plasma.

Plane waves

It is interesting to take the plane wave limit of $AdS_5 x\ S_5$.

We pick a massless geodesic that goes around a maximum circle of the five-sphere. Let us call its momentum J. It is quantized since we are moving along a circle. J is an SO(2) generator inside SO(6), which is the group of rotations of the five-sphere.

The plane wave limit consists in looking only at the region of spacetime that is very near this massless geodesic.

(Penrose, Guven)

We need to take $R \rightarrow$ infinity, keeping

$$\frac{J}{R^2} \quad \text{fixed}$$

and keeping only states with finite E-J , where E is the energy in AdS.

Strings on plane waves

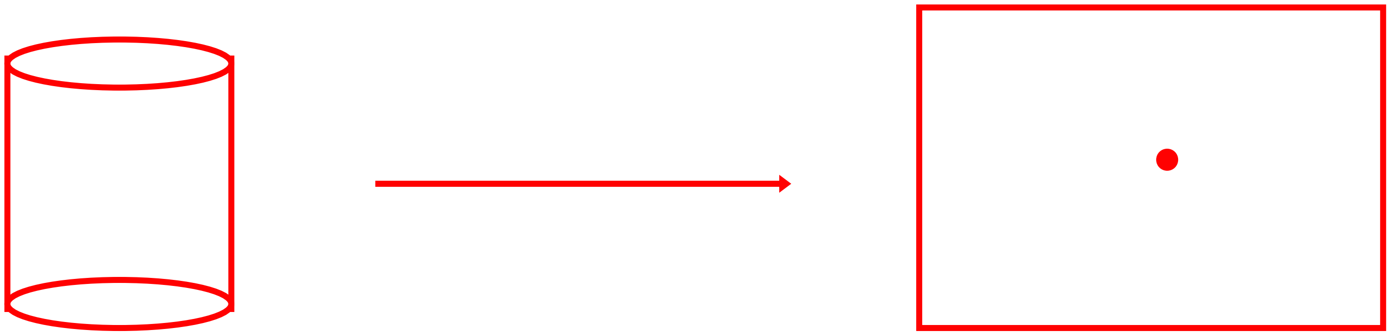
In this limit the resulting spacetime is a plane wave

$$ds^2 = dx^+ dx^- + r^2 (dx^+)^2 + d\vec{r}^{\,2}$$

Strings on plane waves can be quantized exactly. One can find the string spectrum. In light cone gauge one just has 8 massive bosons and fermions. So the string spectrum is given in terms of a massive field theory where Energy of the field theory is Δ-J and the length of the circle is proportional to J.

These are particular states in the AdS space.

The corresponding states in the Yang Mills theory on the three-sphere can be conveniently expressed in terms in terms of operators of the Yang Mills theory on R^4



States on the cylinder $\rightarrow$ Operators on the plane

Energy $\rightarrow$ Conformal weight Δ

In the field theory we take the limit N $\rightarrow$ infinity keeping

Δ -J and J^2/N fixed

The states in the field theory are created by operators of the form

$Tr[Z^J]$ → String in its ground state

$Tr[\Phi Z^J]$ → String in its ground state oscillating around the previous trajectory

$\sum_l Tr[\Phi Z^l \Psi Z^{J-l}] e^{i2\pi n l/J}$ → String with oscillators exited

Phases depend on the position of the "impurities".

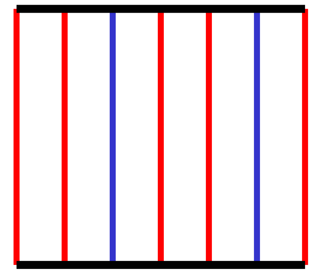
Berenstein, J.M., Nastase

The Zs are "string bits", they define a lattice. The other operators move on This lattice due to the interactions in the YM theory. Planarity is related to Locality along this "string of Zs". The anomalous dimensions can be computed At strong 't Hooft coupling and they agree with the string theory analysis.

Santambrogio, Zanon

Free Yang Mills theory:

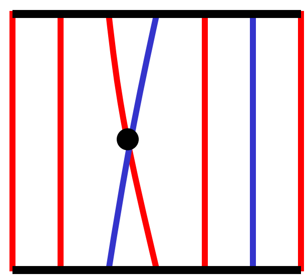
$Tr[\Phi Z^l \Phi Z^{J-l}]$



$Tr[\overline{\Phi}\overline{Z}^l \overline{\Phi}\overline{Z}^{J-l}]$

Interacting theory:

$Tr[\Phi Z^l \Phi Z^{J-l}]$



$g^2 N$

$Tr[\overline{\Phi}\overline{Z}^l \overline{\Phi}\overline{Z}^{J-l}]$

First order correction:

$$\Delta - J = 2 + \frac{g^2 N}{J^2} n^2 \longrightarrow 2\sqrt{1 + \frac{g^2 N}{J^2} n^2}$$ = string theory result.

Is there a dS/CFT ?

Euclidean conformal field theory

= ?

Asymptotic future

Initial singularity

The wavefunction of the universe in the far future region of de-Sitter space would be equal to the partition function of a Euclidean conformal field theory.

(Witten, Strominger,….)

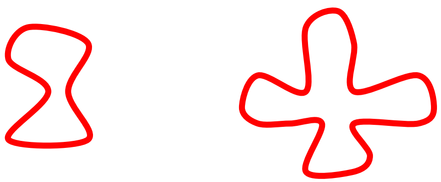
No explicit example is known.
What is the property of the CFT such that the RG direction become timelike in the bulk ?

Future

- Further studies of black holes. Description of the interior of black holes.
- Understand quantum gravity in other spacetimes, especially time-dependent cosmological spacetimes.
- (Large N theories that are closer to the theory of strong interactions, QCD).

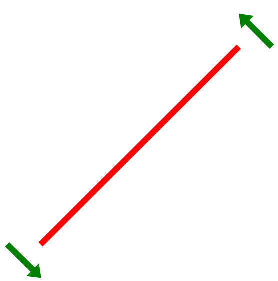
Strings and QCD

In the sixties many new mesons and hadrons were discovered. It was suggested that these might not be new fundamental particles. Instead they could be viewed as different oscillation modes of a string.



This model explained “Regge trajectories”

$$J_{\max} \sim \alpha' m^2 + const$$



Rotating String model

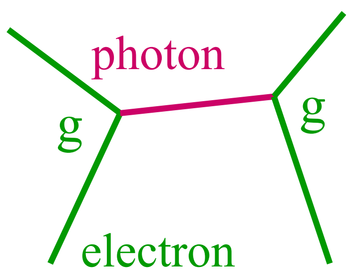
$$m^2 \sim TJ_{\max} + const$$

Other experiments showed that strong interactions are described in terms of quarks and gluons.

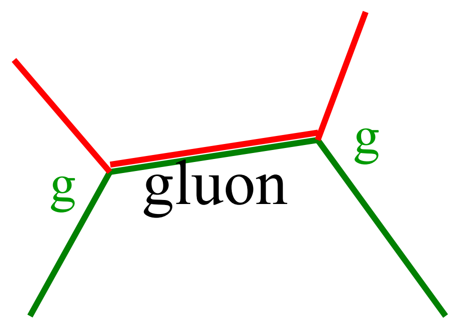
3 colors (charges)

They interact exchanging gluons

Electrodynamics



Chromodynamics

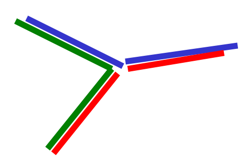


Gauge group

U(1)

SU(3)

3 x 3 matrices



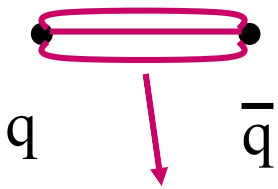
Gluons carry color charge, so they interact among themselves

Coupling constant depends on the energy

$g \longrightarrow 0$ at high energies $\longrightarrow$ QCD is easier to study at high energies

Hard to study at low energies

Indeed, at low energies we expect to see confinement



Flux tubes of color field = glue

$$V = T\ L$$

At low energies we have something that looks like a string

Can we have an effective theory in terms of strings?

t' Hooft Large N limit

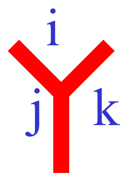
t' Hooft '74

Take N colors instead of 3, SU(N)

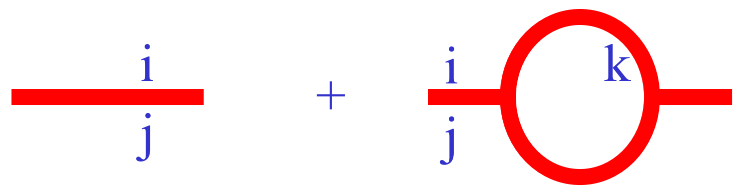
‘t Hooft Limit

Gluon propagator

Interactions

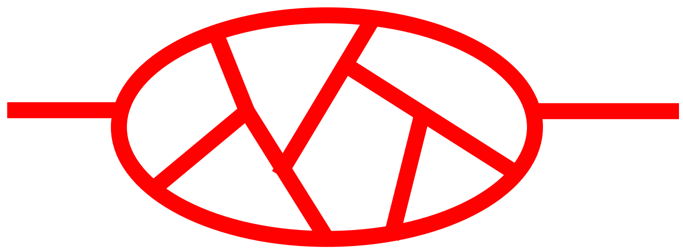


Corrections



1 + g^2N

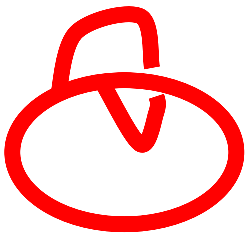
Planar corrections give factors of $(g^2N)^{\text{power}}$



Sum of all planar diagrams will give us a general function $f(g^2N)$

Limit: Keep g^2N fixed and take N to infinity

Non-planar diagrams are suppressed by powers of $1/N$



Thinking of the planar diagrams as discretizing the worldsheet of a string, we see that if g^2N becomes of order one we recover a continuum string worldsheet.

The string coupling constant is of order $1/N$

This might be a good approximation to QCD at low energies, when the coupling is large.

Closed strings would be glueballs.

Open strings would be the mesons.

What is the precise form of the continuum worldsheet action for this string?

Problems:

1) Strings do not make sense in 4 (flat) dimensions

Trying to quantize a string in four dimension one can see that quantum anomalies create at least one more dimension, ie., we need at least one more field along the string to describe its state. (Polyakov)

2) Strings always include a graviton, ie., a particle with m=0, s=2

For this reason strings are normally studied as a model for quantum gravity.

Most Supersymmetric Yang Mills Theory

Supersymmetry

Bosons $\longleftrightarrow$ Fermions

Gluon $\longleftrightarrow$ Gluino

Many supersymmetries

B1 $\longleftrightarrow$ F1
B2 $\longleftrightarrow$ F2

Maximum 4 supersymmetries

A_μ	Vector boson	spin = 1
Ψ_α	4 fermions (gluinos)	spin = 1/2
Φ^I	6 scalars	spin = 0

All NxN matrices

Susy might be present in the real world but spontaneously broken at low energies.
We study this case because it is simpler.

Similar in spirit to QCD

Difference: $\mathcal{N}$ =4 Yang Mills is scale invariant

Classical electromagnetism is scale invariant

$$V = 1/r$$

QCD is scale invariant classically but not quantum mechanically, g(E)

$\mathcal{N}$ =4 Yang Mills is scale invariant even quantum mechanically

<u>Symmetry group</u>

Lorentz + translations + scale transformations + other

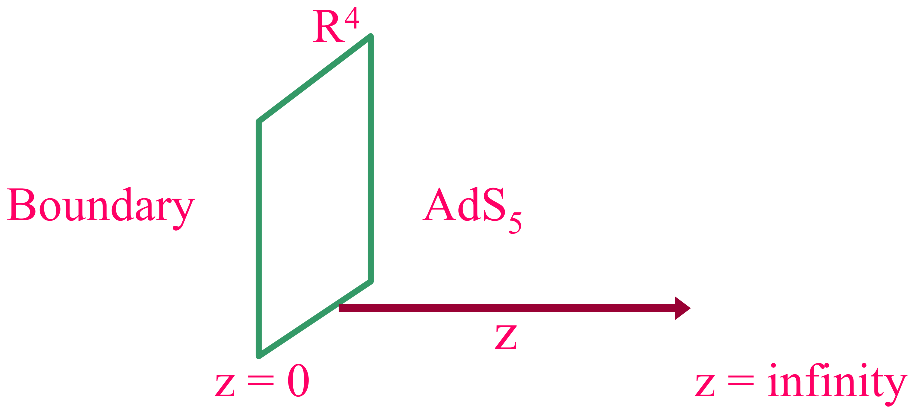
The string should move in a space of the form

$$ds^2 = R^2 \underbrace{w^2}_{\text{redshift factor = warp factor}}(z) \left(dx^2_{3+1} + dz^2 \right)$$

Demanding that the metric is symmetric under scale transformations

$x \rightarrow \lambda x$, we find that $w(z) = 1/z$

$$ds^2 = R^2 \frac{(dx^2_{3+1} + dz^2)}{z^2}$$



This metric is called anti-de-sitter space. It has constant negative curvature, with a curvature scale given by R.

This Yang Mills theory has a large amount of supersymmetry, the same as ten dimensional superstring theory on flat space.

→We add an S^5 so that we have a ten dimensional space.

$$\mathbf{AdS_5 \times S^5}$$

String Theory

Free strings

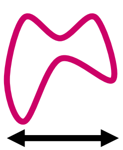
String

$$\text{Tension} = T = \frac{1}{l_s^2} \quad , \qquad l_s = \text{string length}$$

Relativistic, so T = (mass)/(unit length)

Excitations along a stretched string travel at the speed of light

Closed strings



l_s

Can oscillate $\longrightarrow$ Normal modes $\longrightarrow$ Quantized energy levels

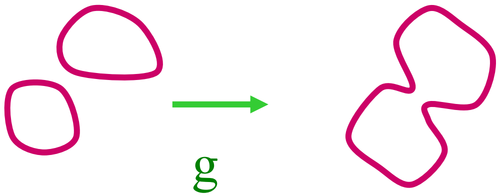
Mass of the object = total energy

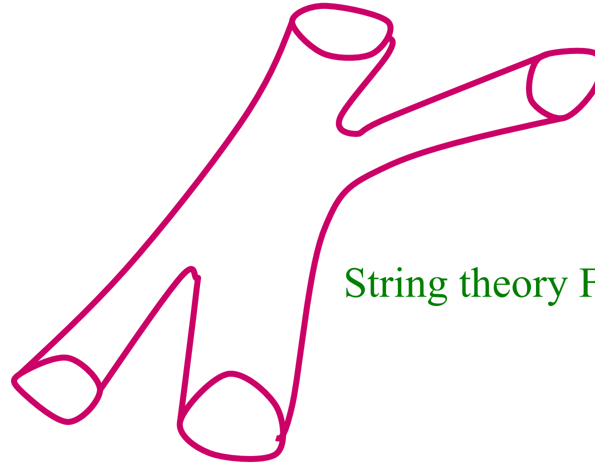
$M=0$ states include a graviton (a spin 2 particle)

First massive state has $M^2 \sim T$

String Interactions

Splitting and joining





String theory Feynman diagram

Simplest case: Flat 10 dimensions and supersymmetric

Precise rules for computing the amplitudes that yield finite results

At low energies, energies smaller than the mass of the first massive string state

↓

Gravity theory

Very constrained mathematical structure

Non-perturbative Aspects

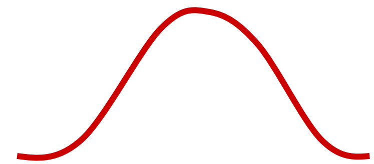
In field theories we can have solitons

e.g. magnetic monopoles (monopoles of GUT theories)

Collective excitations that are stable (topologically)

$$M = \frac{1}{g^2}$$

g = coupling constant



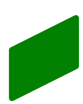
In string theory

we have D-p-branes

Can have different dimensionalities

p=0  D-0-brane   D-particle

p=1  D-1-brane   D-string

p=2  D-2-brane   membrane

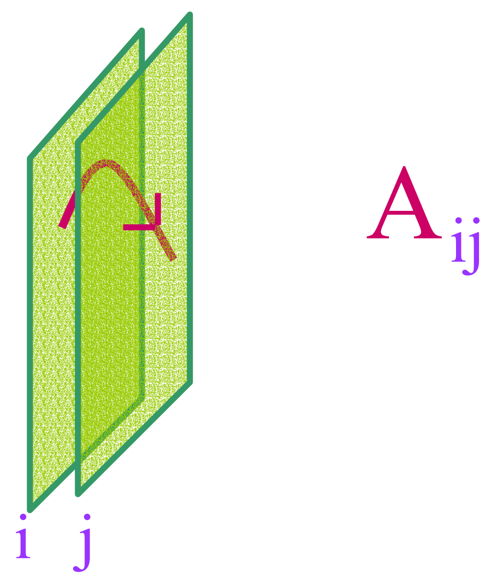
etc.

D-branes have a very precise description in string theory.

Their excitations are described by open strings ending on the brane.

At low energies these lead to fields living on the brane. These include gauge fields.

N coincident branes give rise to U(N) gauge symmetry.

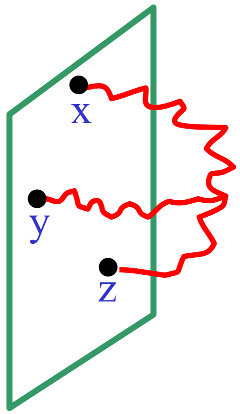


How Do We Use This?

We would like to do computations of the Yang Mills theory at strong coupling, then we just do computations in the gravity theory

Example:
Correlation functions of operators in the Yang Mills theory, eg. stress tensor correlator

$$< T_{\mu\nu}(x)\ T_{\mu\nu}(y)\ T_{\mu\nu}(z) >_{\text{Field theory}} =$$ Probability amplitude that gravitons go between given points on the boundary



Gubser, Klebanov, Polyakov - Witten

Other operators

⟶ Other fields (particles) propagating in AdS.

Mass of the particle ⟶ scaling dimension of the operator

$$\Delta = 2 + \sqrt{4 + (mR)^2}$$

Where Do the Extra Dimensions Come From?

$3+1 \longrightarrow AdS_5 \longrightarrow$ radial dimension

Boundary

z

$1/z \sim$ energy scale

$z=0$

$z=$infinity

ultraviolet,
high energies

infrared,
low energies

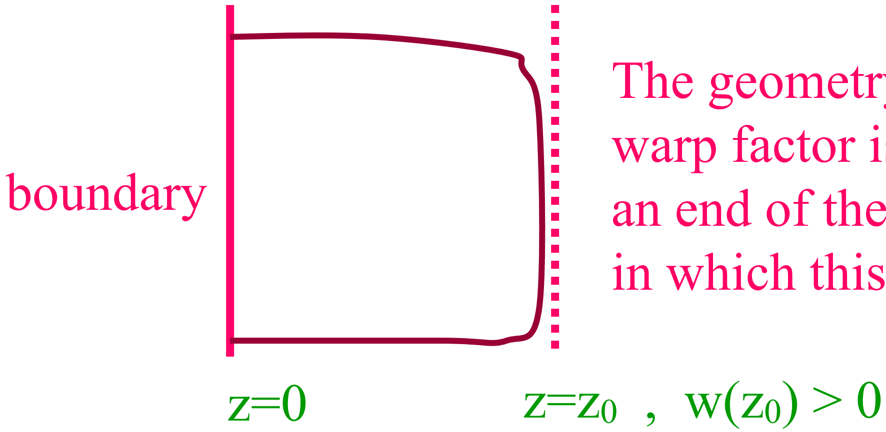
Renormalization group flow: Motion in the radial direction

Five-sphere: is related to the scalars and fermions in the supersymmetric Yang-Mills theory. For other theories the sphere is replaced by other manifolds, or it might even not be there.

Confining Theories

We can add masses to the scalars and fermions so that at low energies we get a pure Yang-Mills theory. At strong coupling it is possible to find the corresponding gravity solution.

There are various examples of theories with one supersymmetry that are confining.



The geometry ends in such a way that the warp factor is finite. We can think of this as an end of the world brane. There are various ways in which this can happen.

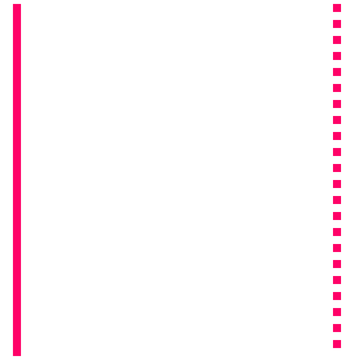
Now the string cannot decrease its tension by going to a region with very small redshift factor. Similarly the spectrum of gravity excitations has a mass gap.

Deconfinement and Black Holes

For these confining theories we can raise the temperature. Then we will find two phases:

At low temperatures we just have a gas of gravitons (strings) in the geometry we had for T=0.

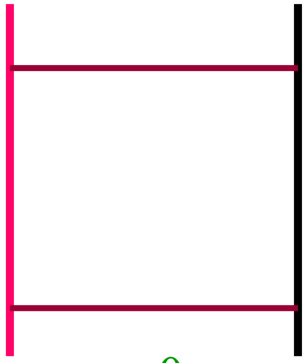
boundary



$S \sim 1$

At high temperatures a black hole (a black brane) horizon forms.

boundary

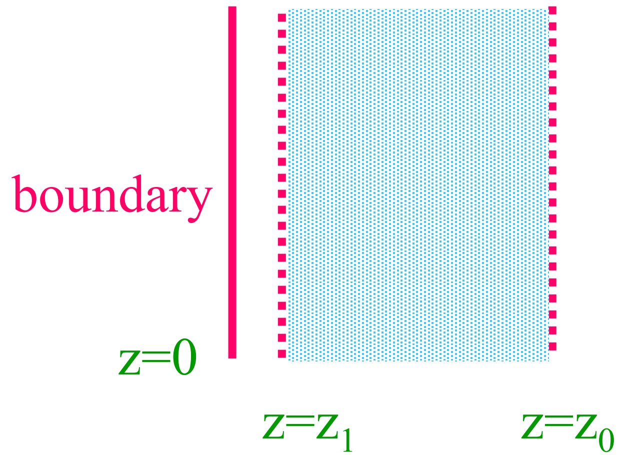


z=0

Horizon. Here the redshift factor $g_{00}=0$.

$$S = \frac{\text{Area of horizon}}{4\, G_N} \sim N^2$$

Randall-Sundrum Models



We only consider a portion of the space with $z_1 < z < z_0$. Cutting off the region with $z<z_1$ is equivalent to introducing a UV cutoff in the field theory, if we keep the metric on the surface $z=z_1$ fixed. Letting this metric fluctuate we are coupling four dimensional gravity.

The RS models are equivalent to 4D gravity coupled to a conformal (or conformal over some energy range) field theory.

Holography

t' Hooft
Susskind

It has been suggested that all quantum theories of gravity should be *holographic.*

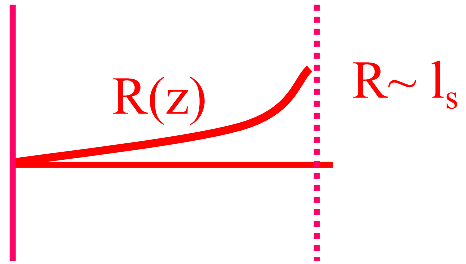
This means that we should be able to describe all physics within some region in terms of a theory living on the boundary of the region, and this theory on the boundary should have less than one degree of freedom per Planck area.

The AdS/CFT conjecture is a concrete realization of this holographic principle

The AdS/CFT conjecture gives a non-perturbative definition of quantum gravity in AdS spaces.

Speculations About Pure Yang Mills

In the large N limit of pure Yang Mills (no susy) we expect to find a string theory on five dimensional geometry as follows:



Near the boundary the AdS radius goes to zero logarithmically (asymptotic freedom). When R(z) is comparable to the string length the geometry ends.

Adding quarks corresponds to adding D-branes extended along all five dimensions. The open strings living on these D-branes are the mesons.

A D0 brane in the interior corresponds to a baryon.

<u>Challenges</u>: 1) Find the precise geometry
2) Solve string theory on it