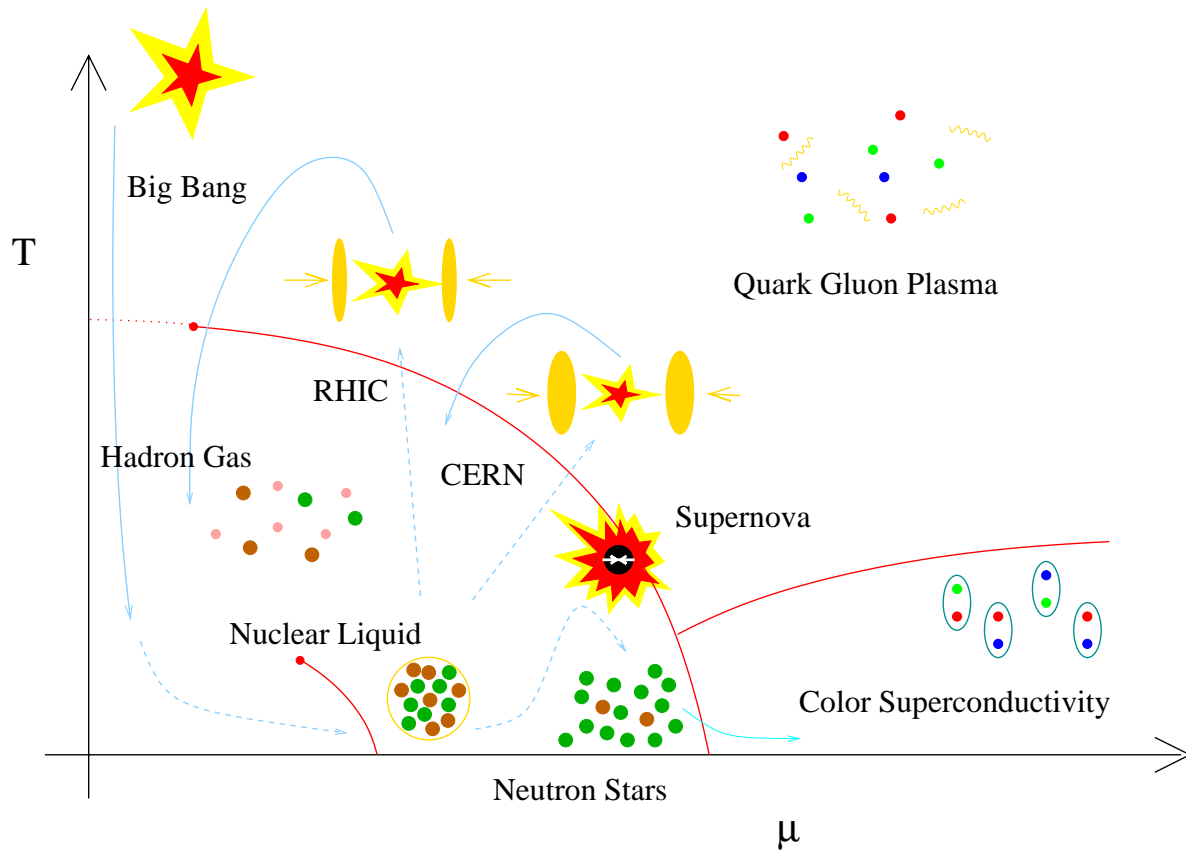


High Density QCD

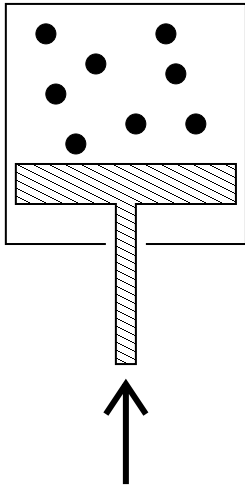
Thomas Schaefer

*North Carolina State University
and Riken BNL Research Center*

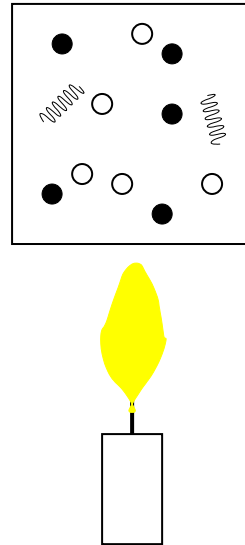


QCD in Xtreme Conditions

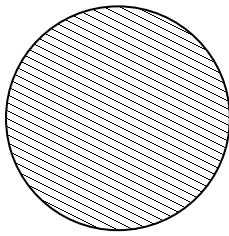
High Density



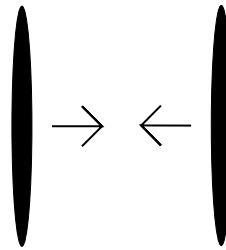
High Temperature



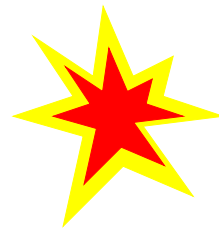
more practical:



neutron
stars



relativistic
heavy-ion
collisions



big bang

WHY?

- These conditions exist in the universe:

Neutron Stars, Big Bang

- Exploring the entire phase diagram is important to understanding the phase that we happen to live in

We cannot understand the structure of hadrons without understanding the non-trivial QCD vacuum.

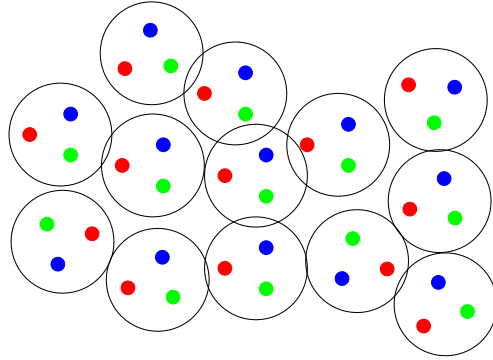
We cannot understand the vacuum state without understanding how it can be modified.

- QCD simplifies in extreme environments:

Study QCD matter in a regime where quarks and gluons are indeed the correct degrees of freedom.

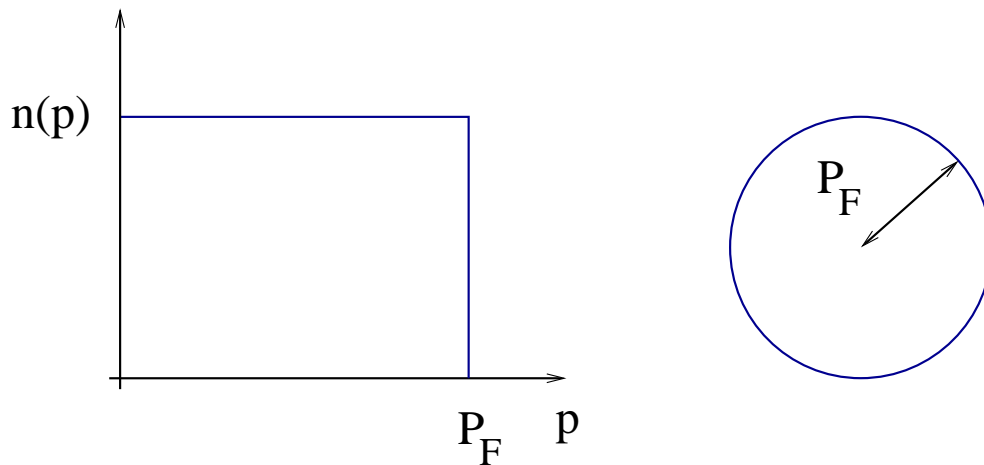
Dense Hadronic Matter

- consider baryon density $n_B \gg 1 \text{ fm}^{-3}$



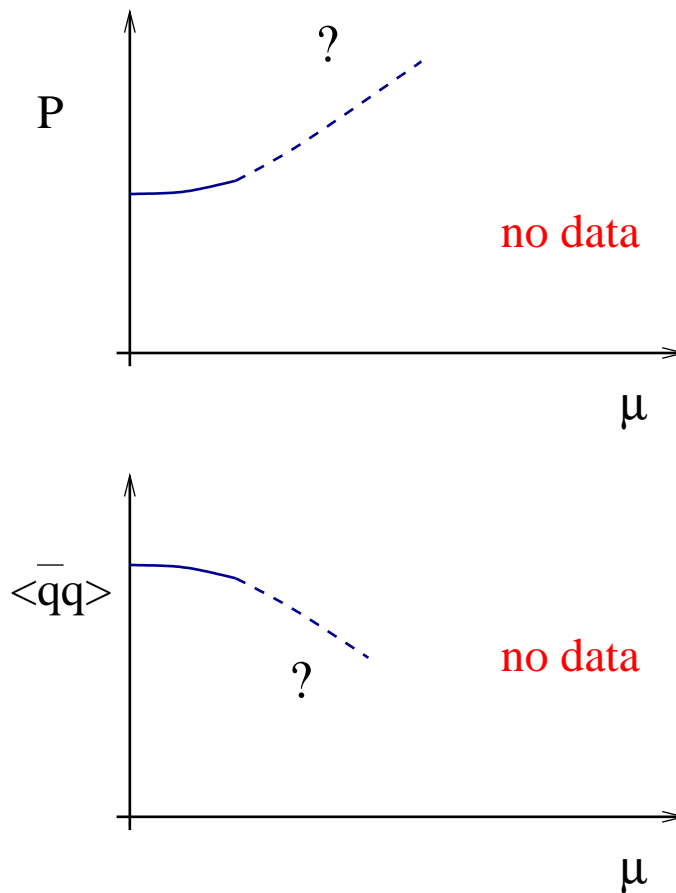
quarks expected to move freely

- groundstate: cold quark matter (quark fermi liquid)



- only quarks with $p \sim p_F \gg \Lambda_{QCD}$ scatter
- $p_F \gg \Lambda_{QCD} \rightarrow$ effective coupling is weak
 - \rightarrow no chiral symmetry breaking
 - \rightarrow no confinement
 - \rightarrow no dynamically generated masses

Lattice Results

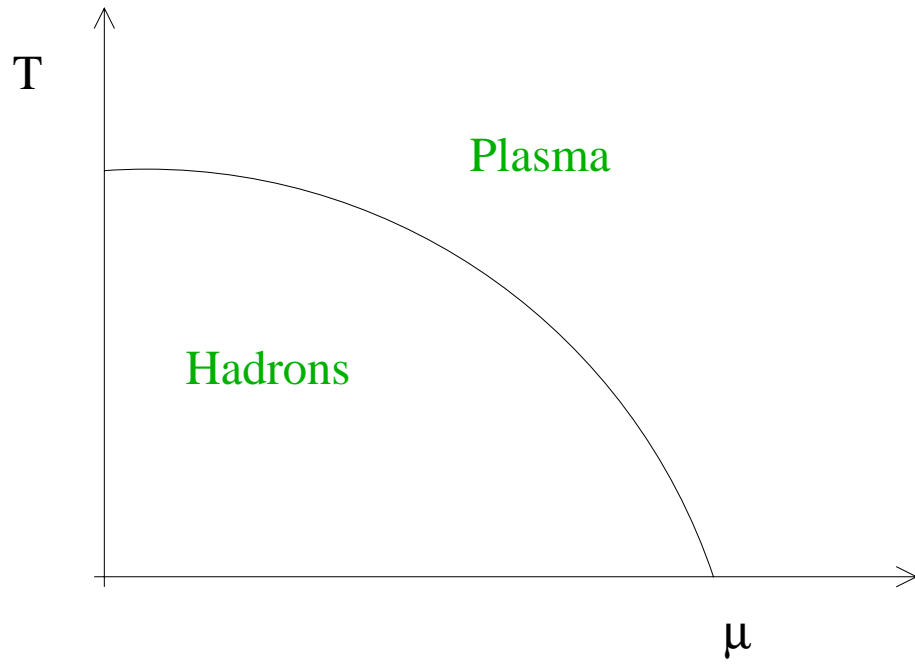


- Problem: No lattice data

$$Z = \int dA \underbrace{\det(\mathcal{D} + i\mu\gamma_4)}_{\text{complex}} e^{-S}$$

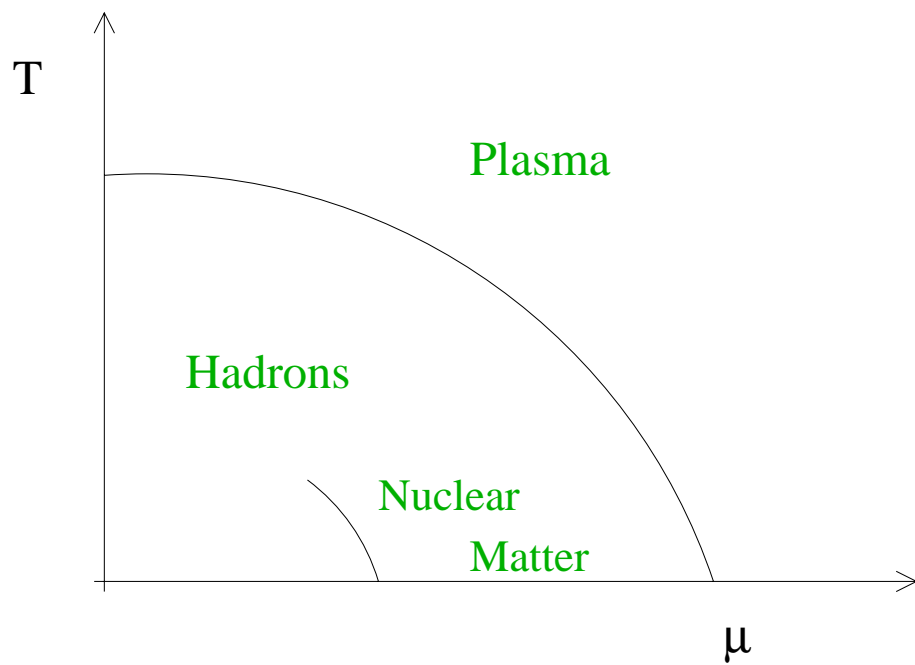
- “sign” problem: no importance sampling
- progress in simulating small μ , $T \sim T_c$
 - improved reweighting (Fodor, Katz)
 - imaginary chemical potential (Forcrand, Philipsen, ...)
 - Taylor expansion (Karsch et al.)

Phase Diagram: First Try



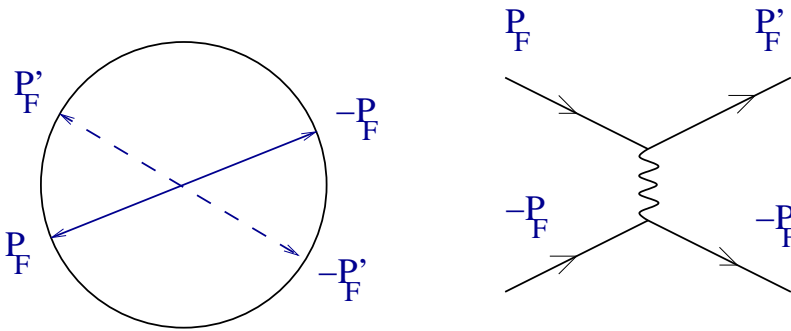
Where are we?

Phase Diagram: Second Try



Superfluidity

- dominant interaction

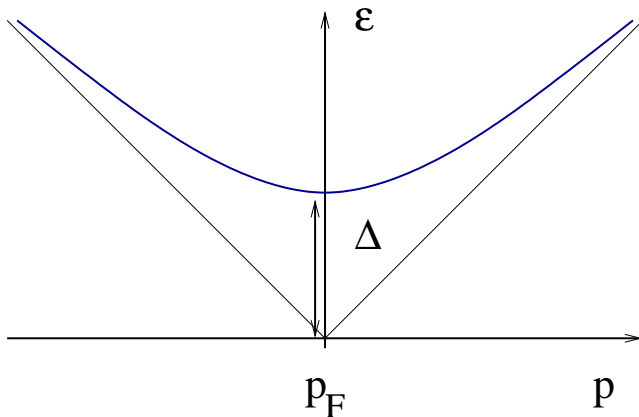


Uses the whole
Fermi surface
coherently

- very degenerate perturbation theory

⇒ instability (BCS) in weak coupling
 ⇒ $q(p_F)q(-p_F)$ condensate

- arbitrarily weak (attractive!) interaction leads to superfluidity/superconductivity (pairs neutral/charged)



$$\epsilon(p) = \sqrt{(p - p_F)^2 + \Delta^2}$$

→ gap in spectrum, transport without dissipation, ...

- QCD: gluon exchange attractive in $\bar{3}$ channel

$$3 \times 3 = 6_S + 3_A \quad \text{Flux reduced} \Rightarrow \text{attractive}$$

- spin-flavor-color wave function

$$(\uparrow\downarrow - \downarrow\uparrow) \times (ud - du) \times (rb - br) \quad s = 0, I = 0, c = \bar{3}$$

order parameter: $\Phi^a = \epsilon^{abc} \langle q^b C \gamma_5 \tau_2 q^c \rangle$

- chiral symmetry is not broken

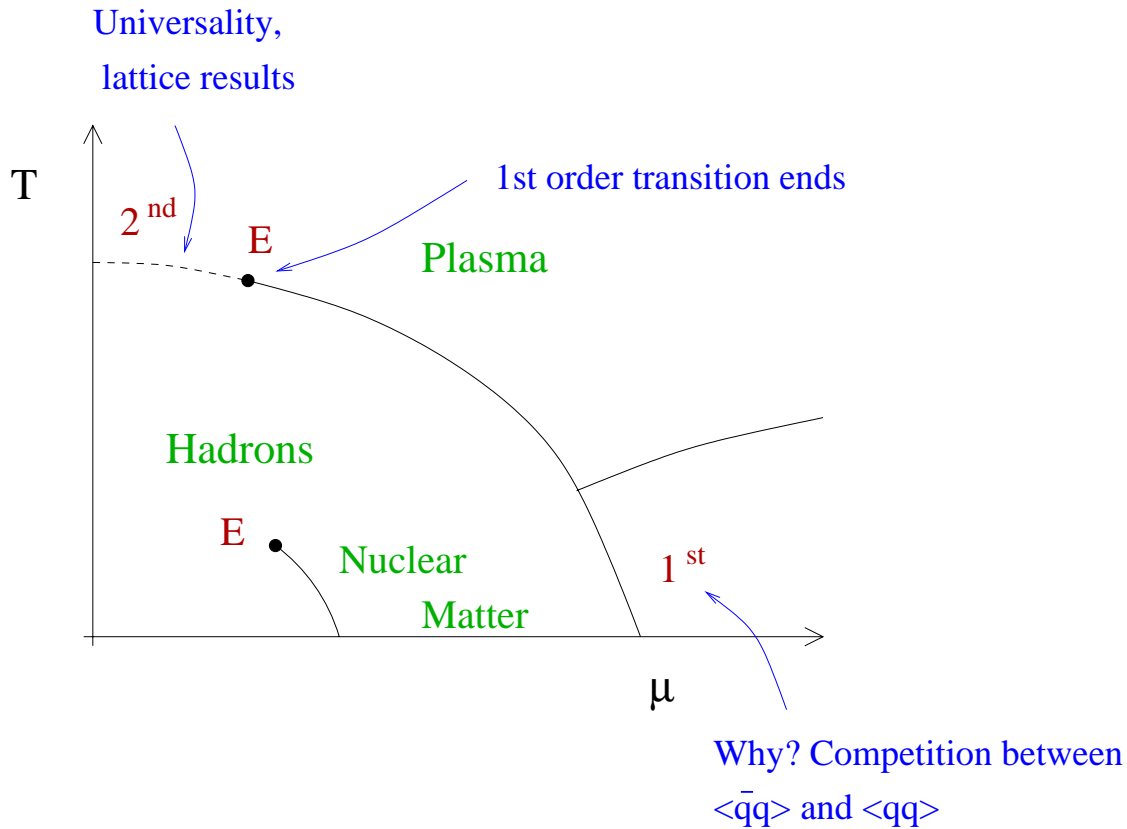
$$\Phi \sim \langle q_L q_L \rangle - \langle q_R q_R \rangle$$

- color symmetry broken by Higgs mechanism (Meissner effect)

$$\Phi^a \in [\bar{3}]; \Rightarrow SU(3) \rightarrow SU(2)$$

- 5/8 gluons acquire mass via Higgs mechanism
- $SU(2)$ is confined

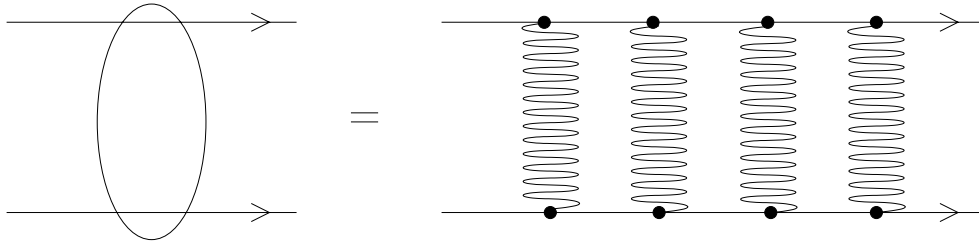
Phase Diagram: Second Revision



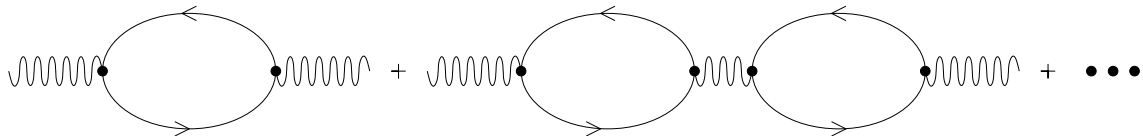
- critical endpoint (E) persists even if $m \neq 0$
- Critical fluctuations observable in heavy ion collisions?
 - charge fluctuations
 - baryon number fluctuations
 - p_T fluctuations, ...

Gap Equation

- $\mu \gg \Lambda_{QCD}$: perturbative forces dominate



- gluon exchange: small angle scattering dominates \rightarrow medium effects important



$$D_E = \frac{1}{q^2 + 2m^2} \quad m^2 = \frac{N_f}{4\pi^2} g^2 \mu^2 \quad \text{Debye screening}$$

$$D_M = \frac{1}{q^2 + i\frac{\pi}{2} m \frac{\omega}{q}} \quad \omega < q \quad \text{Landau damping}$$

- consider $\omega \simeq \Delta$. Typical momenta

$$q_E \simeq g\mu, \quad q_M \simeq (g^2 \mu^2 \Delta)^{1/3}$$

Superconductivity driven by magnetic forces!

- Note: Perturbation theory is self-consistent

$$\mu \rightarrow \infty \quad \Rightarrow \quad q_M \gg \Lambda_{QCD}$$

Eliashberg Equation

- retardation important: Eliashberg theory

$$\Delta(p_0) = \frac{g^2}{18\pi^2} \int dq_0 \left\{ \log \left(\frac{b_M}{|p_0 - q_0|} \right) + \dots \right\} \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

colinear log BCS log

- double logarithmic behavior

$$\Delta_0 \sim \frac{g^2}{18\pi^2} \Delta_0 \left[\log \left(\frac{\mu}{\Delta_0} \right) \right]^2$$

$$\rightarrow \Delta_0 \sim \exp \left(-\frac{c}{\sqrt{\alpha_s}} \right)$$

(BCS result $\Delta_0 \sim \exp \left(-\frac{c}{\alpha_s} \right)$)

- more careful analysis

$$c = 3\pi^2 / \sqrt{2}$$

- condensation energy

$$\epsilon \simeq N \Delta_0^2 \left(\frac{\mu^2}{4\pi^2} \right)$$

- typical numbers

$$\Delta_0 \simeq 100 \text{ MeV} \quad (\rho \sim 5\rho_0)$$

$$T_c \simeq 50 \text{ MeV}$$

Beautiful Case: $N_f = 3$

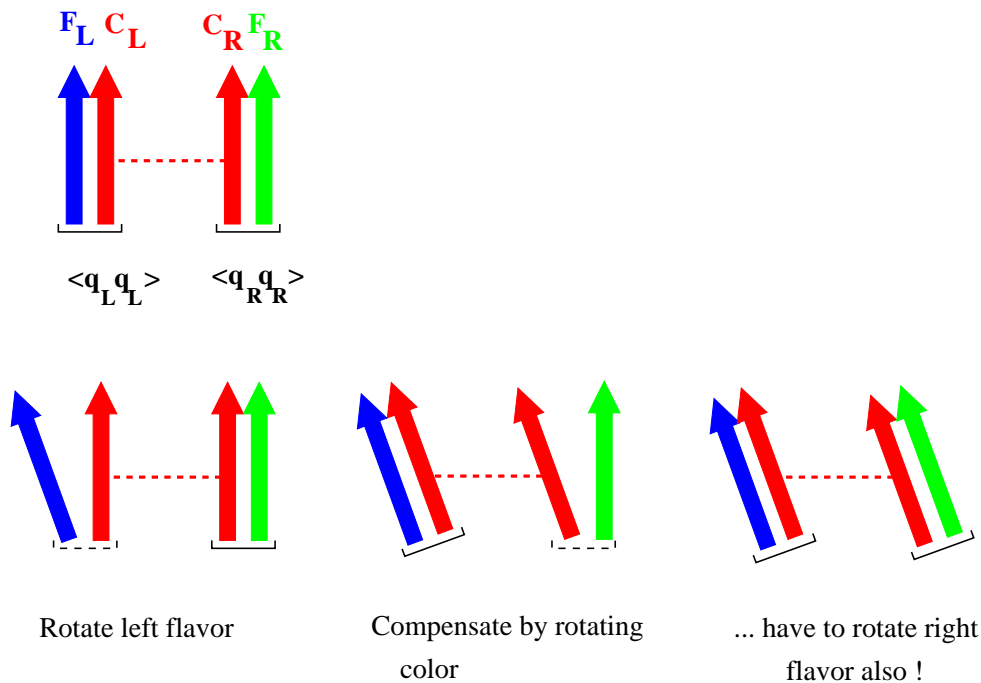
- quarks q_i^a [$i = (u, d, s)$; $a = (r, b, g)$]. Order parameter?

$$\langle q_i^a q_j^b \rangle = \phi \epsilon^{abI} \epsilon_{ijI} \quad \text{Color - Flavor - Locking}$$

$$\begin{aligned} \langle ud \rangle &= \langle us \rangle = \langle ds \rangle && \text{Color and Flavor} \\ \langle rb \rangle &= \langle rg \rangle = \langle bg \rangle && \text{direction aligned} \end{aligned}$$

- Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \rightarrow SU(3)_{C+F}$$



- Novel mechanism for Chiral Symmetry Breaking:

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

Breaks chiral $SU(3)_L \times SU(3)_R$ symmetry because

$$SU(3)_L \xleftrightarrow{\text{Lock}} SU(3)_C \xleftrightarrow{\text{Lock}} SU(3)_R$$

- All quarks and gluons acquire a mass gap

CFL Phase: Excitations

- quark pairs are charged, but $U(1)_{EM}$ remains unbroken

$$\gamma^* = \alpha_{11}\gamma + \alpha_{12}g_3 + \alpha_{13}g_8$$

Material is a transparent insulator

- excitations

$$[8] \quad Q = 0, \pm 1 \quad \text{Goldstones } (\pi, K, \eta) \quad (QQ)(QQ)^{-1}, \dots$$

$$[8] + [1] \quad Q = 0, \pm 1 \quad \text{baryons } (p, n, \Lambda, \Sigma) \quad (Q)(QQ), \dots$$

$$[8] \quad Q = 0, \pm 1 \quad \text{vectors } (\rho, K^*, \dots) \quad g, QQ^{-1}, \dots$$

$$[1] \quad Q = 0 \quad U(1)_A \text{ Goldstone } (\eta') \quad QQ^{-1}, (QQ)(QQ)^{-1}, \dots$$

$$[1] \quad Q = 0 \quad U(1)_B \text{ Goldstone} \quad QQ^{-1}, (QQ)(QQ)^{-1}, \dots$$

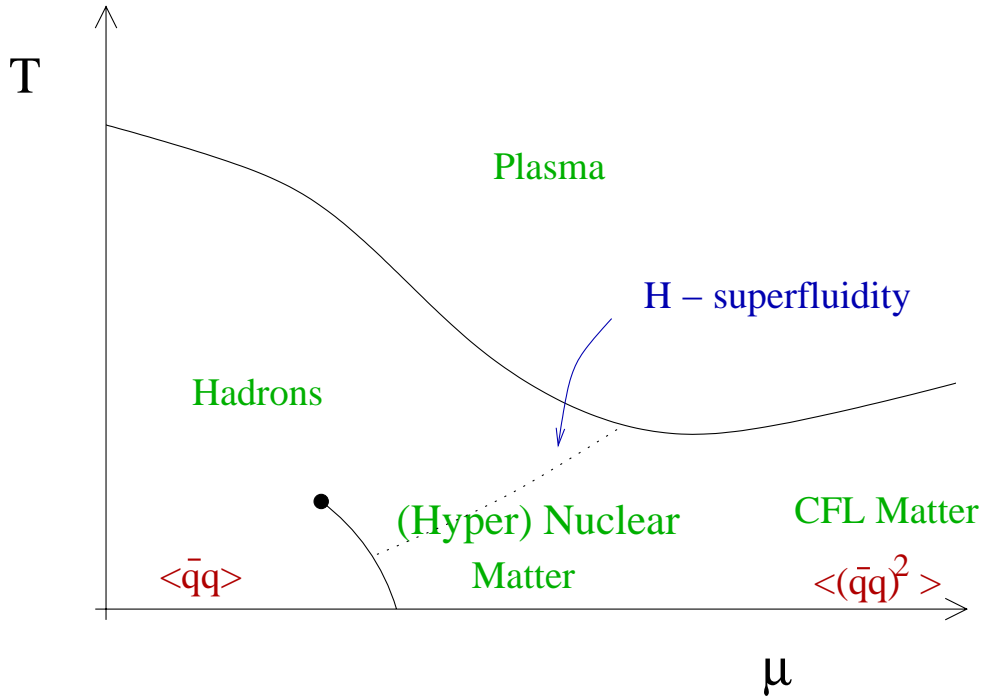
- order parameters

$$\langle \bar{q}q \rangle \quad \text{chiral symmetry breaking}$$

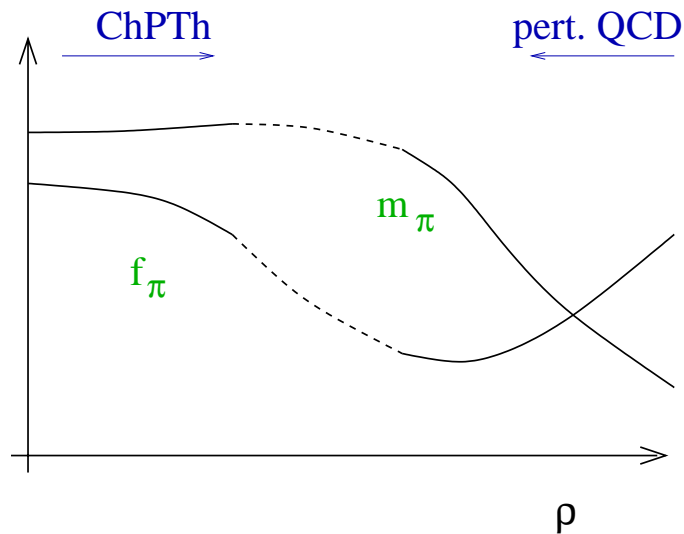
$$\langle (uds)(uds) \rangle \quad \text{H superfluidity}$$

Continuity between quark and hadron matter?

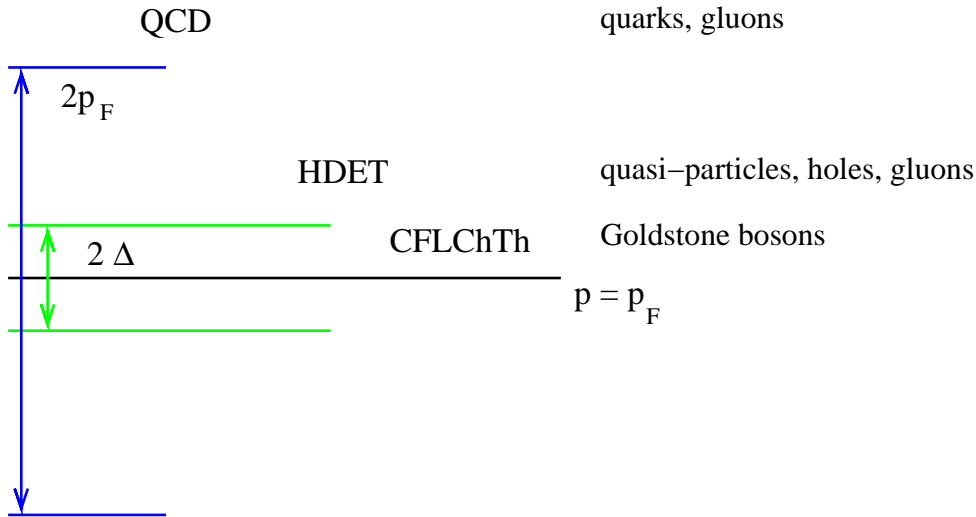
Phase Diagram: Third Revision



Density dependence of hadronic parameters calculable at both low density (effective theory) and high density (perturbative QCD).



Effective Field Theories

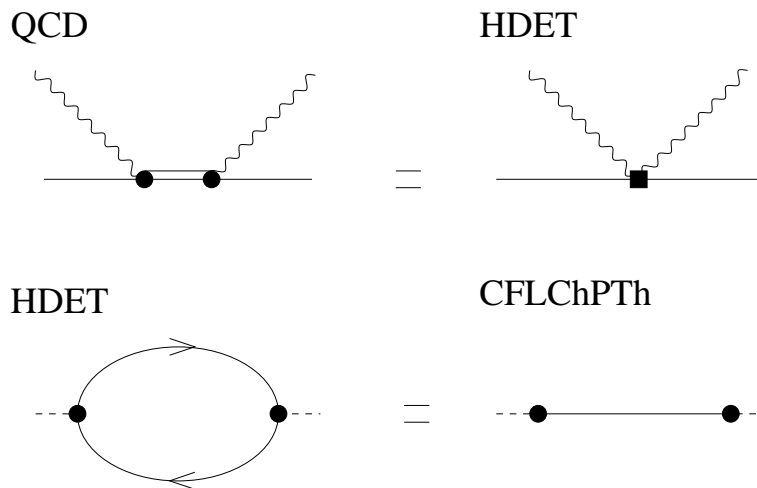


QCD $\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi - \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a$

HDET $\mathcal{L} = \psi_v^\dagger(iv \cdot D)\psi_v - \frac{\Delta}{2}\psi_{-v}^T C\psi_v - \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \dots$

CFLChPTh $\mathcal{L} = \frac{f_\pi^2}{4}\text{Tr} \left[\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \vec{\nabla} \Sigma \vec{\nabla} \Sigma^\dagger \right] + \dots$

- coefficients determined by matching Green functions



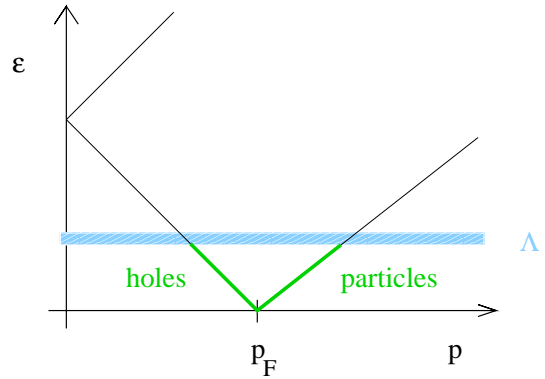
- Applications:
 - f_π, m_π, \dots (collective modes)
 - phase structure as a function of m_s, μ_e, \dots
 - transport properties, ...

High density effective theory

- quasi-particles (holes)

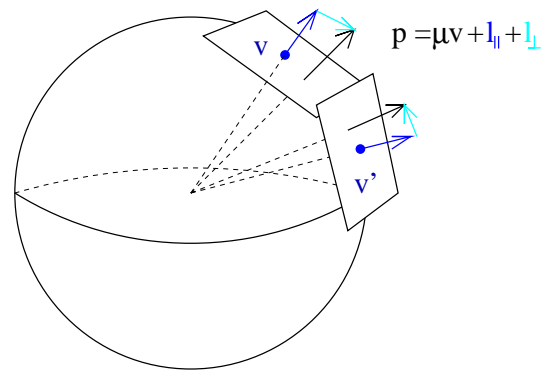
$$E_{\pm} = -\mu + \sqrt{\vec{p}^2 + m^2}$$

$$\simeq -\mu \pm |\vec{p}|$$



- effective field theory on v -patches

$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left(\frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$

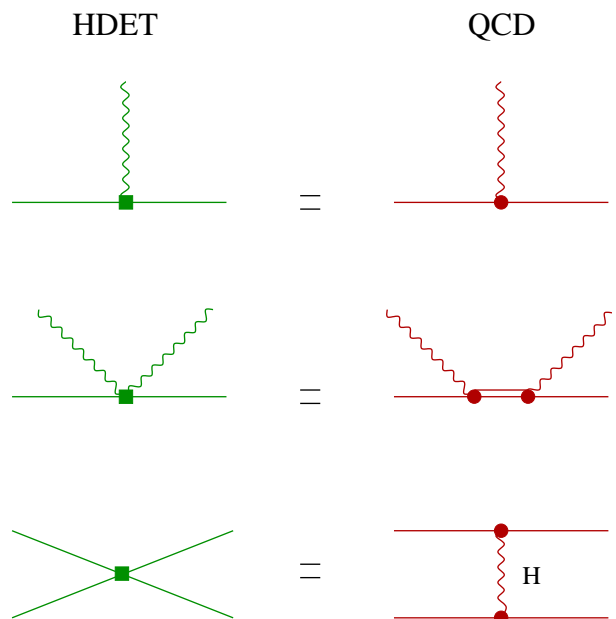


- effective lagrangian for ψ_{v+}

$$\mathcal{L} = \psi_v^\dagger (i v \cdot D) \psi_v$$

$$+ \frac{1}{2p_F} \psi_v^\dagger [(\vec{\alpha}_\perp \cdot \vec{D})^2 + M M^\dagger] \psi_v$$

$$+ \frac{\Gamma_{\vec{v} \cdot \vec{v}'}}{p_F^2} (\psi_{v'}^\dagger \psi_v) (\psi_v^\dagger \psi_{v'}) + \dots$$

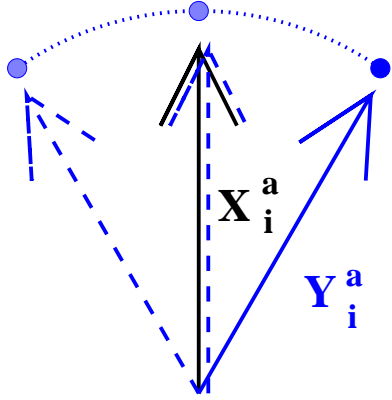


- superconducting phase:

$$\mathcal{L} \rightarrow \mathcal{L} + \psi_v C \Delta \psi_{-v} + \text{h.c.}$$

Effective Chiral Theory

- How to set up a color-flavor wave:



$$X_i^a = \epsilon_{ijk} \epsilon^{abc} \langle (\psi_L)_j^b C (\psi_L)_k^c \rangle$$

$$Y_i^a = \epsilon_{ijk} \epsilon^{abc} \langle (\psi_R)_j^b C (\psi_R)_k^c \rangle$$

$$\langle X_i^a \rangle \sim \delta_i^a$$

$$\langle Y_i^a \rangle \sim \delta_i^a$$

- low energy degrees of freedom

$$\Sigma = XY^\dagger = \exp\left(\frac{i\phi^a \lambda^a}{f_\pi}\right) \quad \phi^a = (\pi, K, \eta, \eta')$$

$$\text{e.g. } K^0 \sim \epsilon^{abc} \epsilon_{ade} (\bar{u}_R^b C \bar{s}_R^c) (d_L^d C u_L^e)$$

- effective theory

$$\begin{aligned} \mathcal{L} = & \frac{f_\pi^2}{4} \text{Tr} \left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \vec{\nabla} \Sigma \vec{\nabla} \Sigma^\dagger \right) \\ & + A \text{Tr}(M \Sigma^\dagger) V e^{i\Theta} \quad \leftarrow U_A(1) \text{ anomaly} \\ & + B_1 [\text{Tr}(M \Sigma^\dagger)]^2 + \dots \end{aligned}$$

- expansion

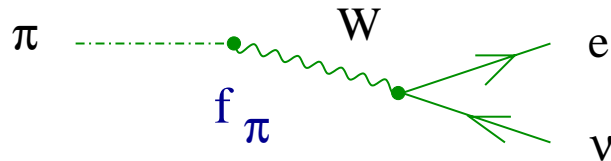
$$\mathcal{L} \sim f_\pi^2 \Delta^2 \left(\frac{\vec{\partial}}{\Delta} \right)^{N_1} \left(\frac{\partial_0 + MM^\dagger/p_F}{\Delta} \right)^{N_2} \left(\frac{MM}{p_F^2} \right)^{N_3} (\Sigma)^{N_4} (\Sigma^\dagger)^{N_4}$$

Note: $\Lambda_{\chi SB} \sim \Delta \ll 4\pi f_\pi$

Matching, part I

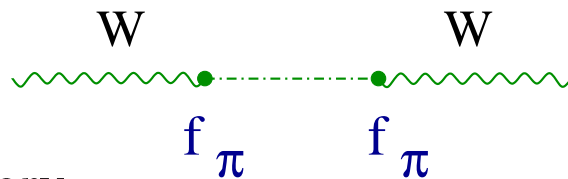
- Compute f_π : Gauge $SU(3)_L \times SU(3)_R$ flavor symmetry

$$\partial_\mu \Sigma \rightarrow \nabla_\mu \Sigma = \partial_\mu \Sigma - iW_\mu^L \Sigma + i\Sigma W_\mu^R$$

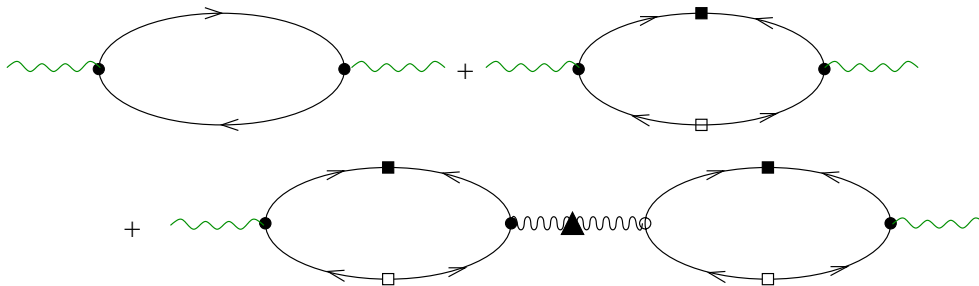


- Higgs phenomenon

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} [(W_0^L - W_0^R)^2] + \dots \quad m_W^2 = f_\pi^2$$



- microscopic theory



$$f_\pi^2 = \frac{21 - 8 \log(2)}{18} \left(\frac{\mu^2}{2\pi^2} \right)$$

- Note: $4\pi f_\pi \sim p_F \gg \Delta$

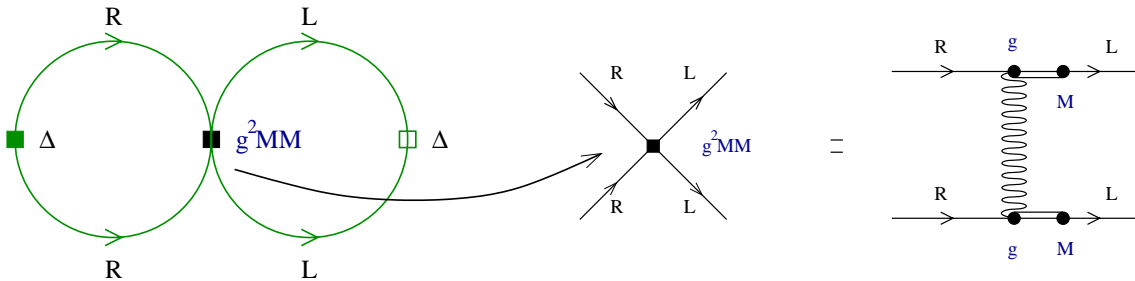
$$\rho \sim 5\rho_0 \Rightarrow p_F \sim 500 \text{ MeV} \Rightarrow f_\pi \sim 100 \text{ MeV}$$

Matching, part II

- Compute m dependence of vacuum energy

$$\Delta\mathcal{E} = -B_1 [\text{Tr}(M)]^2 - B_2 \text{Tr}(M^2) \quad \Sigma = 1$$

- microscopic theory



$$\begin{aligned} \Delta\mathcal{E} &\sim \left(\frac{g^2}{p_F^4}\right) [p_F^2 \Delta \log(\Delta)]^2 \{[\text{Tr}(M)]^2 - \text{Tr}(M^2)\} \\ &\sim \Delta^2 \{[\text{Tr}(M)]^2 - \text{Tr}(M^2)\} \end{aligned}$$

$$B_1 = -B_2 = \frac{3\Delta^2}{4\pi^2}$$

- meson masses

$$\begin{aligned} m_\pi^2 &= \frac{3\Delta^2}{4f_\pi^2} (m_u + m_d) m_s \\ m_{K^\pm}^2 &= \frac{3\Delta^2}{4f_\pi^2} (m_u + m_s) m_d \end{aligned}$$

- Note:

$$m_{GB} \sim 10 \text{ MeV} \quad m_K < m_\pi$$

Matching, part III

- consider $1/p_F$ expansion

$$\mathcal{L} = \psi_L^\dagger \left(p_0 - \epsilon_p - \frac{MM^\dagger}{2p_F} \right) \psi_L + \frac{\Delta}{2} \psi_L C \psi_L \\ + (L \leftrightarrow R, M \leftrightarrow M^\dagger) + O(1/p_F^2)$$

- MM^\dagger and $M^\dagger M$ enter as gauge fields

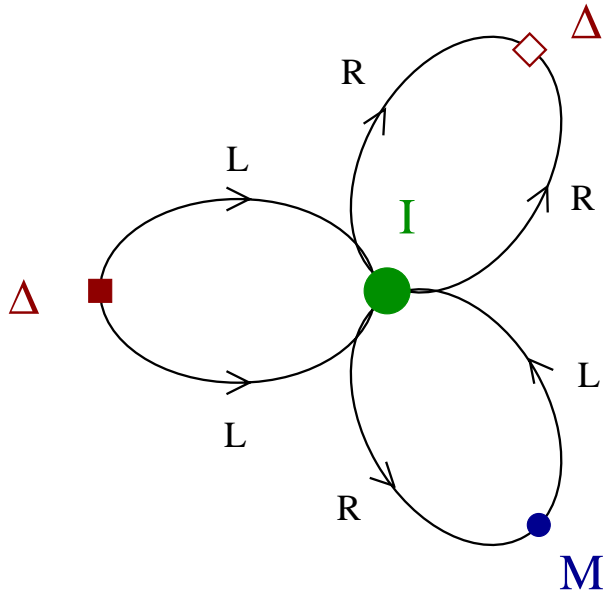
$$W_L = \frac{MM^\dagger}{2p_F} \quad \psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R \\ W_R = \frac{M^\dagger M}{2p_F} \quad W_L \rightarrow LW_L L^\dagger + iL\partial_0 L^\dagger, \dots$$

- implement gauge symmetry in effective lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} \left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger \right) + \dots \\ \nabla_0 \Sigma = \partial_0 \Sigma + i \frac{MM^\dagger}{2p_F} \Sigma - i \Sigma \frac{M^\dagger M}{2p_F}$$

Matching, anomalous part

- linear term $\text{Tr}(M\Sigma)$ in vacuum energy related to instantons (violates $(Z_2)_A$ symmetry)



Instanton size

$$\rho \sim \mu^{-1} \ll \Lambda_{QCD}^{-1}$$

- instanton contribution to vacuum energy: $\mathcal{E} = A\text{Tr}(M)$

$$A = C_{N_c}^{N_f} \langle \psi C \psi \rangle^2 \left(b \log \left(\frac{\mu}{\Lambda_{QCD}} \right) \right)^6 \left(\frac{\Lambda_{QCD}}{\mu} \right)^1 2\Lambda_{QCD}^{-3} e^{i\Theta}$$

$$\langle \psi C \psi \rangle = \frac{3\sqrt{2}\pi}{g} \Delta \left(\frac{\mu^2}{2\pi^2} \right)$$

- Note: $\langle \bar{\psi} \psi \rangle = -A$

$$\langle \bar{\psi} \psi \rangle \sim \left(\frac{\Lambda_{QCD}}{\mu} \right)^8 \Lambda_{QCD}^{-3} \ll \Lambda^{-3}$$

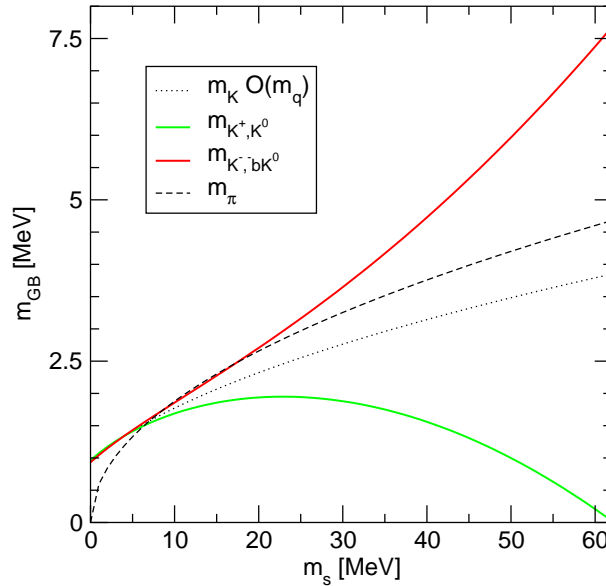
Kaon condensation

- K^0 mass vanishes if

$$m_s > m_s|_{crit}$$

$$m_s|_{crit} \simeq 3m_u^{1/3} \Delta^{2/3}$$

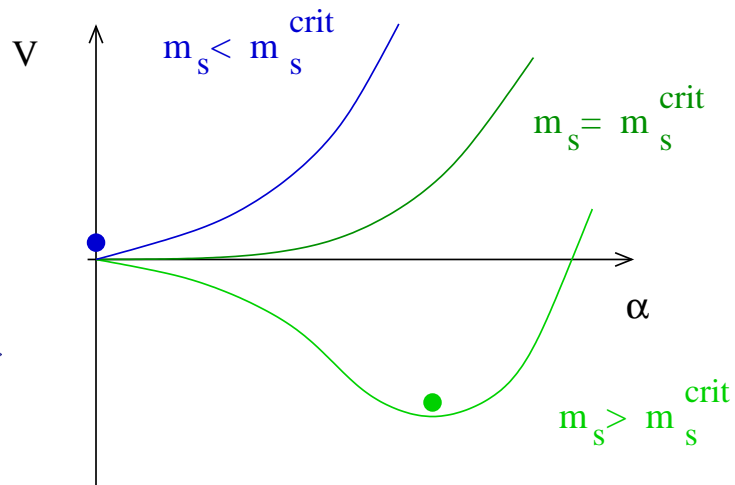
⇒ kaon condensation



- new vacuum state:

$$\Sigma = \exp(i\alpha\lambda_4)$$

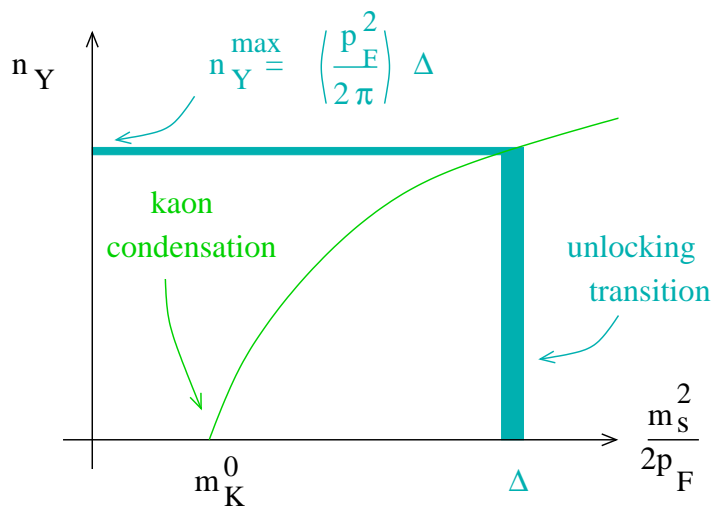
$$V(\alpha) = -f_\pi^2 \left\{ \frac{m_s^2}{4p_F} \sin^2(\alpha) + (m_K^{(0)})^2 (\cos(\alpha) - 1) \right\}$$



- hypercharge density

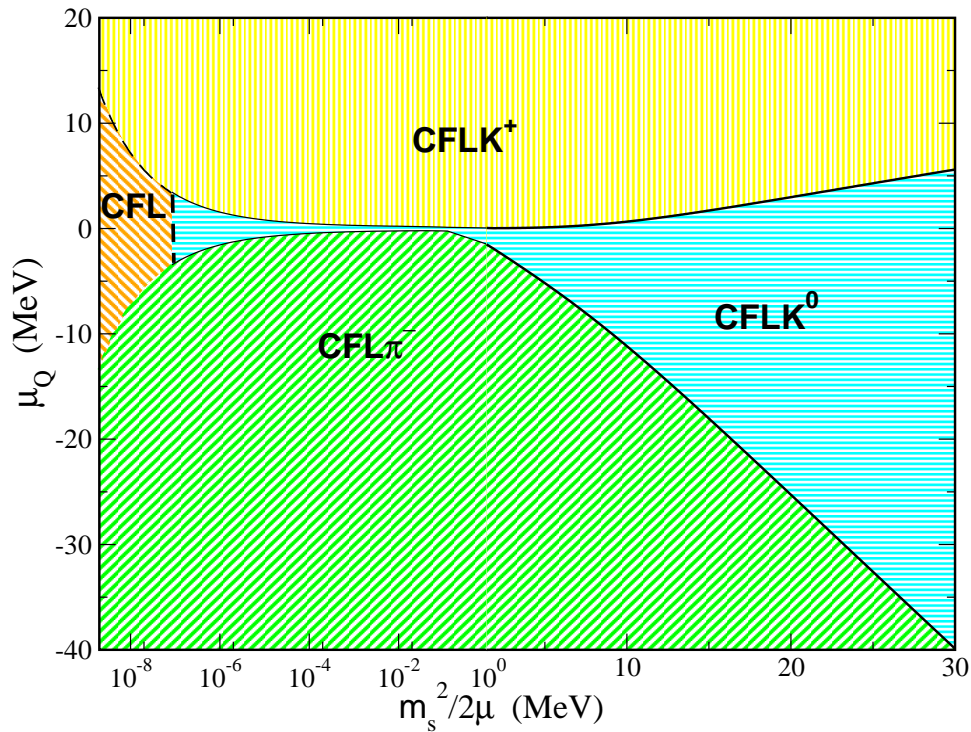
$$n_Y = f_\pi^2 \mu_{eff} \left\{ 1 - \frac{(m_K^{(0)})^4}{\mu_{eff}^4} \right\}$$

$$\mu_{eff} = m_s^2 / (2p_F)$$



Phase Diagram of CFL phase

$$V(\Sigma) = \frac{f_\pi^2}{2} \text{Tr} (W_L \Sigma W_R \Sigma^\dagger) - A \text{Tr} (M \Sigma^\dagger) - B_1 [\text{Tr} (M \Sigma^\dagger)]^2 + \dots$$



- response to masses/ext. fields determined by collective states

T. Schäfer, P. Bedaque, Nucl. Phys. A697,802,2002

D. Kaplan, S. Reddy, Phys.Rev.D65:054042,2002

- Goldstone modes dominate transport, neutrino emissivity,

Jaikumar, Prakash, Schäfer (2002)

Ellis, Shovkovy (2002)

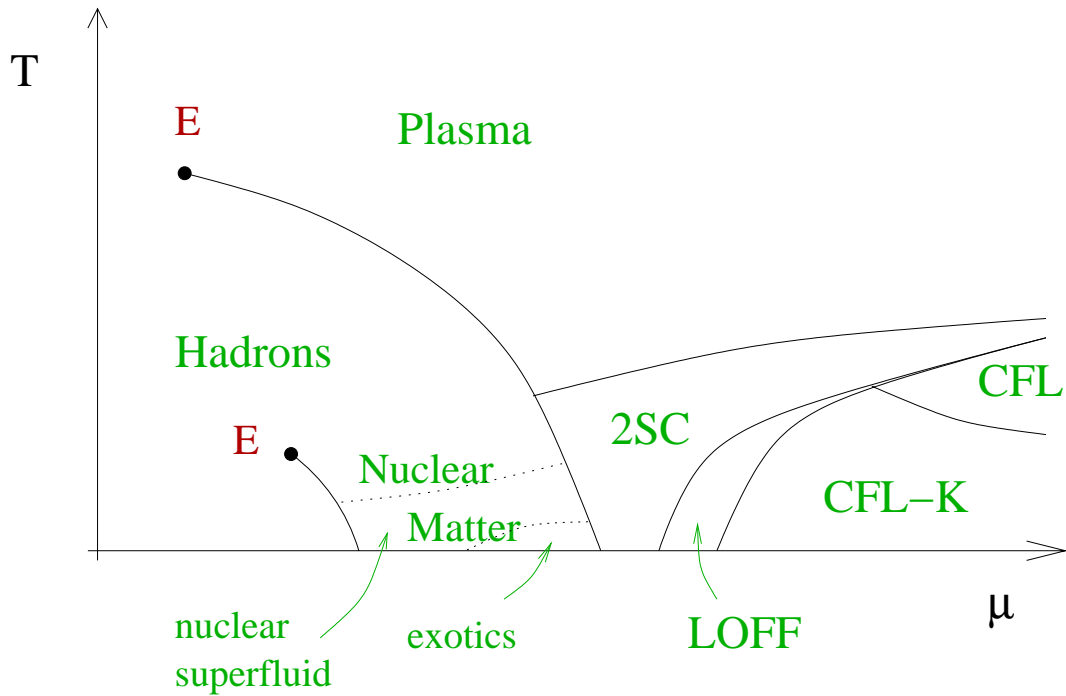
Reddy, Sadzikowski, Tachibana (2002)

- superconducting K -strings

Kaplan, Reddy (2002)

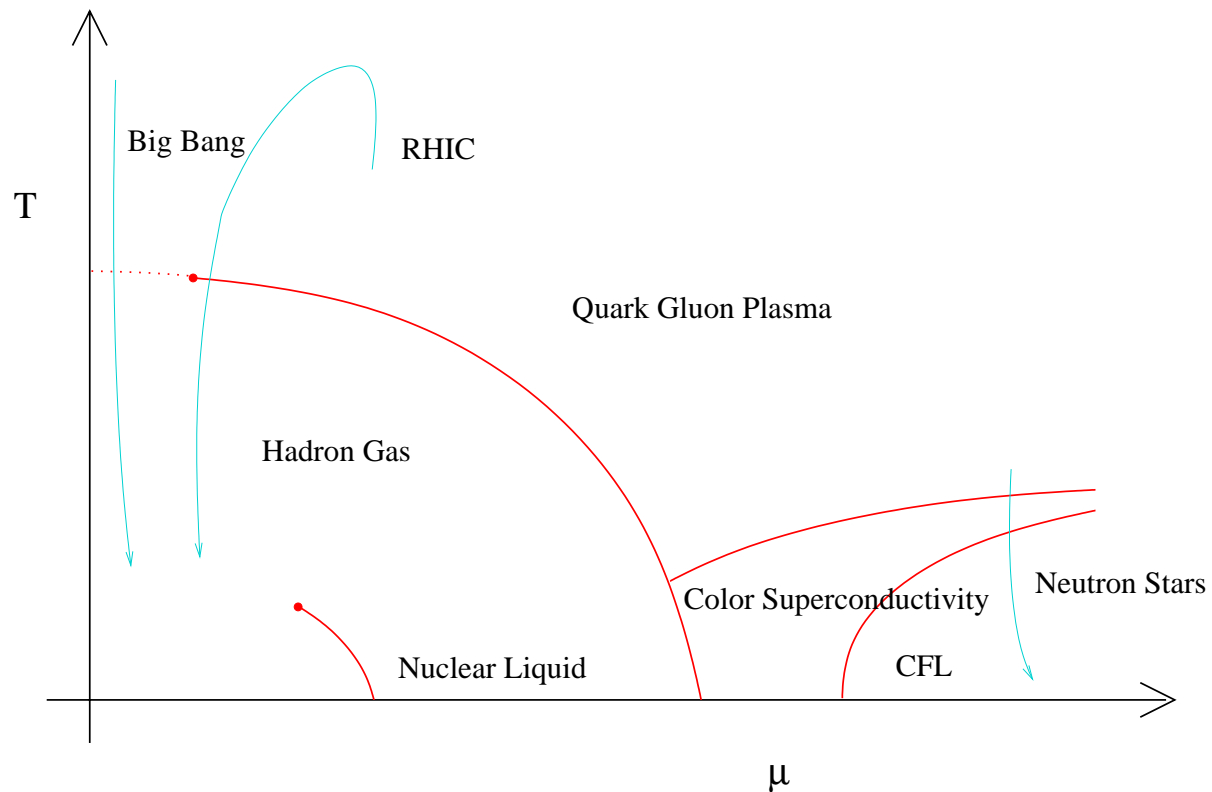
Buckley, Zhitnitsky (2002)

Phase Diagram: $m_s \neq 0$



- phase structure at moderate μ (and $m_s \neq 0$) complicated and poorly understood
- “real world” effects (charge neutrality, neutrino transparency) important
- observables for neutron star phenomenology

Summary: The Many Phases (Faces?) of QCD



- Interesting new phases of QCD at non-zero baryon density: Color Superconductivity, Color-Flavor-Locking
- Theory: Weak coupling realization of chiral symmetry breaking and mass gaps
- Phenomenology: Neutron Stars (cooling, neutrino burst, glitches?), Heavy Ions (Tri-critical point, precursor phenomena)